AN ANALYSIS OF EM WAVE CIRCUIT PROBLEMS BY NETWORK-BOUNDARY ELEMENT METHOD

Feng Zheng-he

Department of Radio and Electronics
Tsing-hua University, Beijing, China

Abstract

The paper describes a new method which combines Boundary Element Method (BEM) with network method and is called "Network-Boundary Element Method". This hybrid method needs small amount of nodes, can get network parameter directly. It can be used to solve planar circuit problems and discontinuous problems of transmission lines. Some examples are given in the paper.

Introduction

A new method which combines network method with Boundary Element Method (BEM) is described in the paper. It needs smaller amount of nodes compared to other numerical method and the network parameters can be got directly.

An arbitrary planar EM circuit can be divided into several subregions according to their boundary or medium conditions. A boundary equation can be set up in each region with own fundamental solution. In port's subregion, the Green's function of transmission line is used, and the field on two end boundaries is expanded in intrinsic functions and boundary elements respectively, so the network parameters can be handled as boundary conditions.

This method can result in a program in common use, which can be used in multi-port, multi-mode, multi-medium arbitrary planar problems and radiation problems.

Summary of theory

1. Fundamental solution:

A 2-D region S enclosed by boundary Γ, is illustrated in Fig.1. A potential function u(x,z) satisfies following equation in S:

$$(\nabla^2+k^2)u(x,z)=0 \qquad \text{in } S \qquad (1)$$

$$u=\bar{u} \qquad \text{in } \Gamma_1$$

$$q=\frac{\partial u}{\partial n}=\bar{q} \qquad \text{in } \Gamma_2 \qquad (2)$$

By Weighted Residual Method, equation (1) can be rewritten as:

$$u_{in}+\int_\Gamma u\frac{\partial\omega}{\partial n}\,d\Gamma=\int_\Gamma \omega\frac{\partial u}{\partial n}\,d\Gamma \qquad (3)$$

where u and q are boundary values on Γ, u_{in} is the values of inside point, ω is the fundamental solution of problem (1). In boundless and irregular boundary problems, Hankel fuction $H_0^{(2)}(k\rho)$ is proper as described in[3].

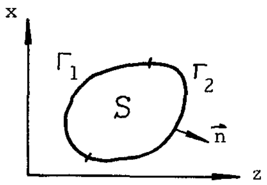

Fig.1 2-D region

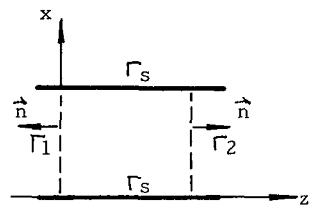

Fig.2 Boundary of a section of waveguide

In this paper, we put stress on the fundamental solution of transmission line. They can be expressed as:

$$\omega(x,z;x',z')=\sum_{n=1}^{\infty}\frac{1}{j\beta_n}\eta_n(x)\eta_n(x')\exp(j\beta_n|z-z'|) \qquad (4)$$

where $\eta_n(x)$ isintrinsic function.

In Fig.2, a section of waveguide is shown, where z is the direction of propagation. The integrals of eqn.(3) on two side walls Γs is zero, so the integral is carried only on $\Gamma 1$, $\Gamma 2$.

When observastion point on inside area goes onto the boundary, the singular problem must be considered. we have verified that when fundamental solution (4) is used, the singular point can be moved and equation (3) becomes:

$$u_\Gamma/2+\int_\Gamma u\frac{\partial\omega}{\partial n}\,d\Gamma=\int_\Gamma \omega\frac{\partial u}{\partial n}\,d\Gamma \qquad (5)$$

where Cauchy's principle value means this integral term is not necessary when observation point and source point is on same boundary $\Gamma 1$ or $\Gamma 2$.

2. Fourier Transform and network parameters:

If $\Gamma 1$ is perpendicular to side boundary Γs as shown in Fig.2. The field on $\Gamma 1$ can be expressed as expansion in series of $\eta_m(x)$:

$$u(x)=\sum_{m=1}^{\infty}a_m(z)\eta_m(x) \qquad (6)$$

$$q(x)=\sum_{m=1}^{\infty}b_m(z)\eta_m(x) \qquad (7)$$

where the infinit summation can be interrupted by some number, for example m=M, M is the highest guide mode of the waveguide. That is to say, only guide mode are considered, and each mode can be regarded as a transmission line.

Substituting these equations into equation(5) and dividing them into two kinds, one equation where observation point is on $\Gamma 1$ and other on $\Gamma 2$.

By using Fourier transform we can get:

$$\frac{1}{2} a_m(z)(1+S_m) - \frac{1}{2}\int_{\Gamma_2} u_2(x')\eta_m(x')dx' * \exp(j\beta_m|z|) = \frac{1}{2}\frac{1}{j\beta_m}\int_{\Gamma_2} q_2(x')\eta_m(x')dx' * \exp(j\beta_m|z|)$$

$$m=1,2,\cdots,M \qquad (8)$$

$$\frac{1}{2} u_2(x,z) - \frac{1}{2}\sum_{m=1}^{M}(1-S_m)a_m(z)\eta_m(x) * \exp(j\beta_m|z|) = \int_{\Gamma_2} q\omega\, d\Gamma \qquad (9)$$

we can define

$$S_m(z) = \frac{a_m(z)}{b_m(z)}\,\frac{1}{j\beta_m} \qquad (10)$$

as a normalized impedance (Z) or conductance (Y). The physical meaning of these parameters are shown in table 1.

be regarded as a boundary condition (3rd boundary condition or impedance condition).

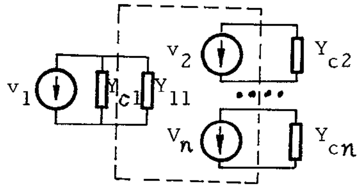

Fig.3 Equivalent circuit of multi-mode waveguide

In Fig.3 is shown the equivalent circuit of LSM mode problem in which the network is expressed in Y parameters, the variables in dash frame are unknown and the others are known.

Table 1 The Network Parameter

mode	u(x)	q(x)	$a_m(z)$	$b_m(z)$	$S_m(z)$	boundary condition	example of transmission line
LSM	Ey	Hx	v	i	Y	1st kind 2nd kind	H-plane waveguide microstrip line
LSE	Hy	Ex	i	v	Z	2nd kind	E-plane waveguide

The integral on $\Gamma 2$ of eqn.(8) and eqn.(9) can be discretized by interpolation. we can divide $\Gamma 2$ into several elements, and use constant, linear or high order interpolation on each element, so u(x) can be expressed as function of nodes. And then the observation points are discretized by nodes, as result we get a set of linear equation. The integal in eqn.(8),(9) can be expressed in a analytical forms, not necessary to do numerical integral, if we use eqn.(4) as fundamental solution.

3. Subregions and matrix equation:

In multi-port, multi-dielectric or unboundary problems, we can divide region S into several subregions. On each subregion, we can set up an integral equation with own fundamental solution, and then discretize them into a set of linear equations. By employing the continuous conditions on the common boundaries between subregions and other known boundary condition which includs network parameter, these equations can be combined into a matrix equation :

$$[A] * \vec{X} + \vec{F} = 0 \qquad (11)$$

where $[A]$ is coefficient matrix, $\vec{F}$ is known vector, $\vec{X}$ is unkown one, the network parameter and boundary value are simultaneously included in these vectors. Equation (11) can be solved by Gaussian elimination method.

Application examples

1. Discontinuous problem in waveguide:

Verious kind of discontinuity of waveguide can be handled by NBEM, such as inductive or capacitive diaphragms, post, junction or displacement of waveguide. For multi-mode guide, each mode can be regarded as a port, or, when calculating each mode can be regarded as a node, as shown in Fig.4-7, the dots on dash line. Indeed, the relation between network parameters, as expressed by eqn.(10), cán

An inductive window in multi-mode waveguide is shown in Fig.4(a).The node is represented by dot • . The results are shown in Fig.4(b).

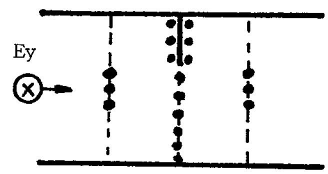

Fig.4(a) Inductive window in multi-mode waveguide

In Fig.5 is shown a capacitive window with thickness, because of symmetry, the sketch of that is only half. The T-type equivalent circuit parameters and measurement results are also shown in the same figure where t-t, t'-t' are reference planes.

The reflective factor of displaced waveguide are shown in Fig.6.

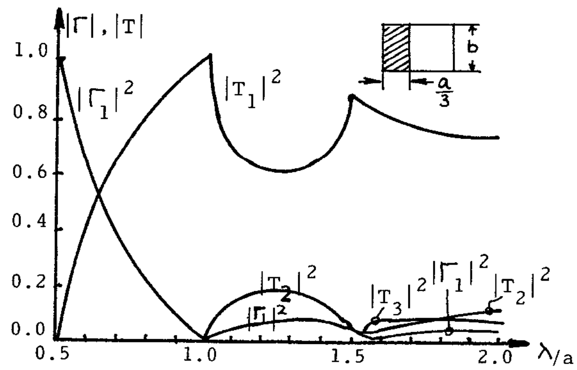

Fig.4(b) Reflection and transmission factor of multi-mode waveguide

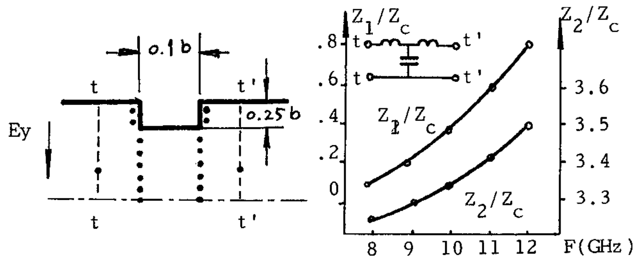

Fig.5 Capacitve window with thickness

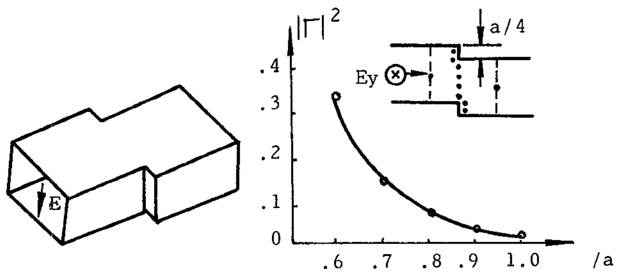

Fig.6 Reflection factor of displaced H-plane waveguide

2.Boundless problem:

The problem of radiation from H-plane waveguide is shown in Fig.7. Because of symmetry , the number of nodes is half reduced, which are expressed in the same figure by dot • . The region is divided into two subregions, one is a section of waveguide and the other is half space where the fundamental solution is $H_0^{(2)}(k\rho)$. The results is shown in Fig.7 too.

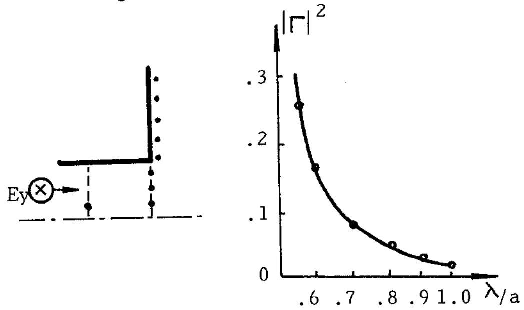

Fig.7 Radiation from H-plane open ended waveguide

3. Planar EM circuit problem:

The example of planar EM circuit is the symmetric 5-port waveguide junction as shown in Fig.8. The region is divided into 4-subregions as shown in same figure. When each port is connected by a matched load, the calculated data are shown in Fig.9.

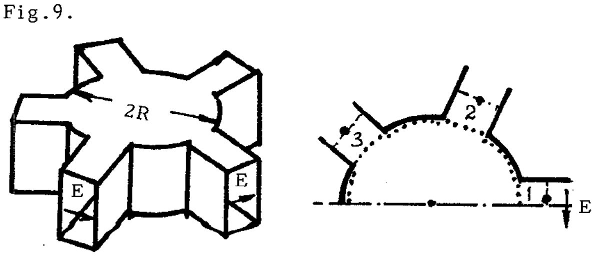

Fig.8 Symmetrical 5-Port waveguide Junction

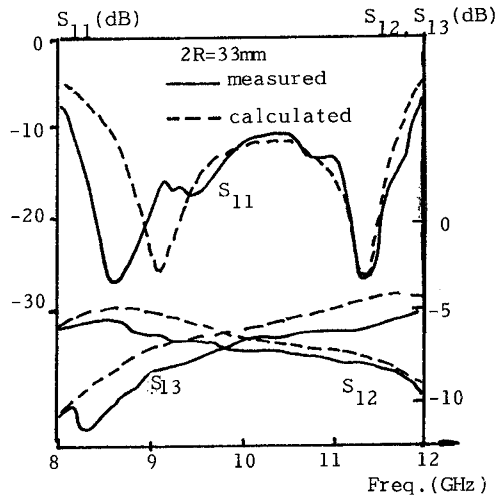

Fig.9 Reflection and transmission factors of 5-port junction with matched loads

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